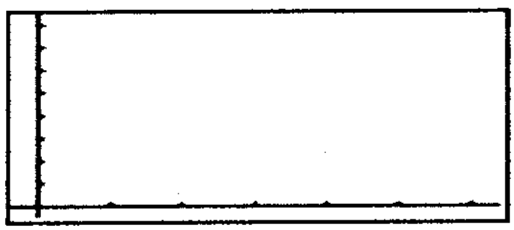
Calculus Application Problem - Fuel It Up!

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a differentiable function R of time t. A table of selected values of R(t), for the time interval $0 \le t \le 90$ minutes is shown below.

t	0	15	30	45	60	75	90
R(t)	20	35	63	78	73	60	56

1. On your calculator, make a **scatterplot** of the ordered pairs in the chart above. Show the scatterplot below and connect the points to show the continuous function $y = R(t)$. (Note: When connecting the points, assume the maximum occurs at the point (45,78)).



2. Using the data from the table, calculate an approximation for $R'(30)$ and $R'(75)$. Show the computations that lead to your answers, and indicate the units of measure.

3. The rate of fuel consumption is increasing the fastest at time t=22.5. What is the value of $R''(22.5)$? ________

Explain your reasoning.____

4. If $A(x) = \int_0^x R(t)\,dt$, answer the following questions about A(x).

a. Where is A(x) concave up? ____________ What is true about R(t) on this interval? ________________

b. Where is A(x) concave down? ____________ What is true about R(t) on this interval? ________________

c. There are no maximum or minimum values on the graph of A(x). Using the graph of $y = R(t)$, explain why this is true.

____

5. For some value b, $0 \le b \le 90$ minutes, explain the meaning of $A(b) = \int_0^b R(t)\,dt$ in terms of fuel consumption of the plane. Indicate the units of measure. ________________________________

____

____

6. Explain the meaning of the expression $\frac{1}{b} \cdot \int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate the units of measure. (Hint: This was a concept covered earlier in this course!) ____________________

____

____

____

7. Using a "right" Riemann sum with the six subintervals indicated by the data in the table, write and evaluate an expression that will approximate the value of $A(90) = \int_0^{90} R(t)\,dt$. Draw these "approximating rectangles" on your graph of R(t).

Would you guess that this approximation is "less than" or "greater than" the actual value of A(90)?. Explain your reasoning. ____

8. Using the Trapezoidal Rule, write and evaluate an expression to approximate the value of $A(90) = \int_0^{90} R(t)\,dt$. Indicate the units of your answer.

9. A student proposes that the function R for the rate of consumption of the fuel for the first ninety minutes, in gallons per minutes, is given by:

$$R(t) = .4t - 20\cos(\pi t/45) + 40$$

Graph R(t) on your calculator with your scatterplot to determine if the student was correct (on $0 \le t \le 90$).

10. Using the function rule for R(t), find R'(30), R'(75), and R''(25). Show your work. Tell how your answers compare with your answers to part 2 and part 3.

11. Using the function rule for R(t), apply the Trapezoidal Rule with 20 subintervals. Wite and evaluate an expression (involving sigma) to approximate the value of $A(90)=\int_0^{90} R(t)dt$.

12. Using the function rule for R(t), write and evaluate an expression to find the <u>exact</u> value for the total number of gallons of gasoline that the airplane consumes over the first ninety minutes of the flight. Show your work.

13. Determine the <u>average rate of fuel consumption</u> for the ninety minute flight. ____________________

14. The <u>Mean Value Theorem for Integrals</u> states that for a continuous and positive function f on a closed interval [a,b], there must exist at least one number c in [a,b], such that:

$$f(c)=\frac{1}{b-a}\cdot\int_a^b f(x)dx$$

Find all values of c in the interval [0,90], which satisfies the Mean Value Theorem for Integrals. Show the equation needed to be solved to find the value of c. (Note: You may find c using any method you choose. That is, you can solve the equation graphically.)